Cellular Automata Models of Traffic Flow

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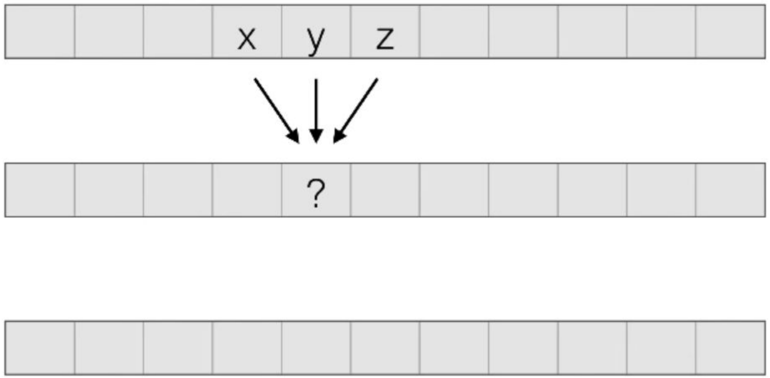
Cellular Automata (CA)

- Discrete mathematical framework to model complex phenomena from simple rules

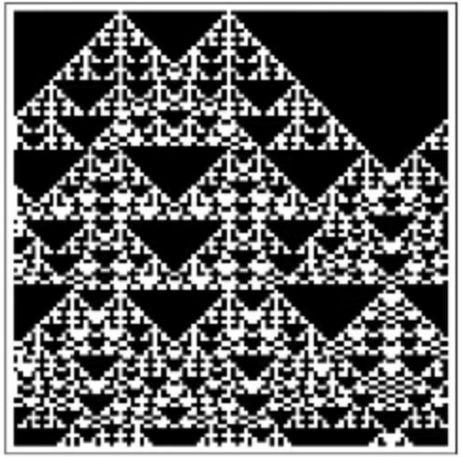
$$t = t - 1$$

$$t = t$$

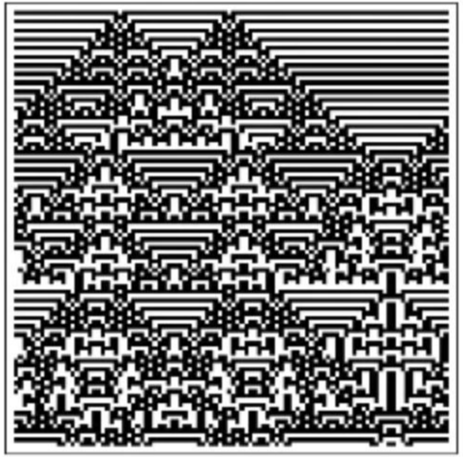
$$t = t + 1$$



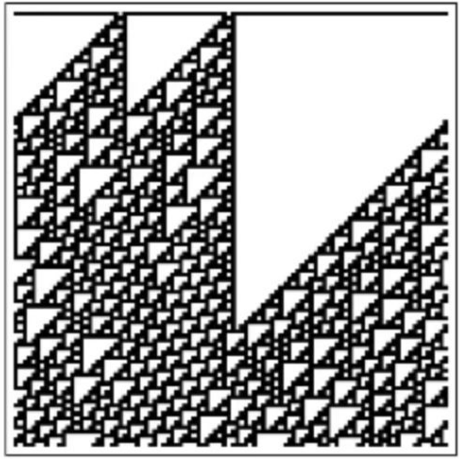
$$state(t) = f(state(t-1)) = f(x, y, z)$$



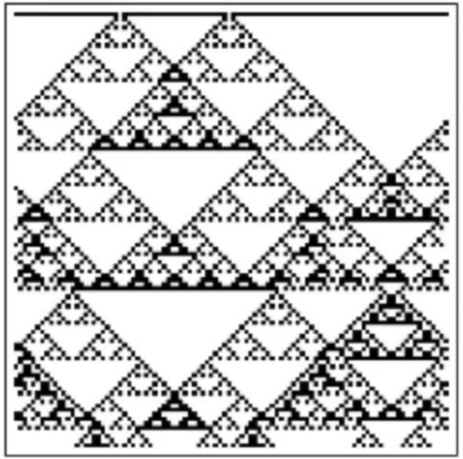
Rule 150



Rule 105



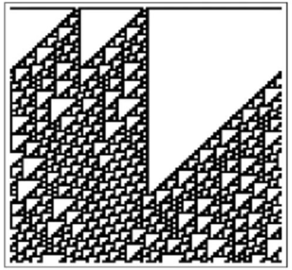
Rule 110



Rule 90

Image credit: Santa Fe Institute

Cool Rules



Rule 110

current pattern	111	110	101	100	011	010	001	000
new state for center cell	0	1	1	0	1	1	1	0

Universality in Elementary Cellular Automata

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The purpose of this paper is to prove a conjecture made by Stephen Wolfram in 1985, that an elementary one dimensional cellular automaton known as "Rule 110" is capable of universal computation. I developed this proof of his conjecture while assisting Stephen Wolfram on research for *A New Kind of Science* [1].

1. Overview

The purpose of this paper is to prove that one of the simplest one dimensional cellular automata is computationally universal, implying that many questions concerning its behavior, such as whether a particular sequence of bits will occur, or whether the behavior will become periodic, are formally undecidable. The cellular automaton we will prove this for is known as "Rule 110" according to Wolfram's numbering scheme [2].

Reference:
https://wpmedia.wolfram.com/uploads/sites/13/2018/02/15-1-1.pdf

Cool Rules

- $$$ in Rule 30: https://writings.stephenwolfram.com/2019/10/announcing-the-rule-30-prizes/

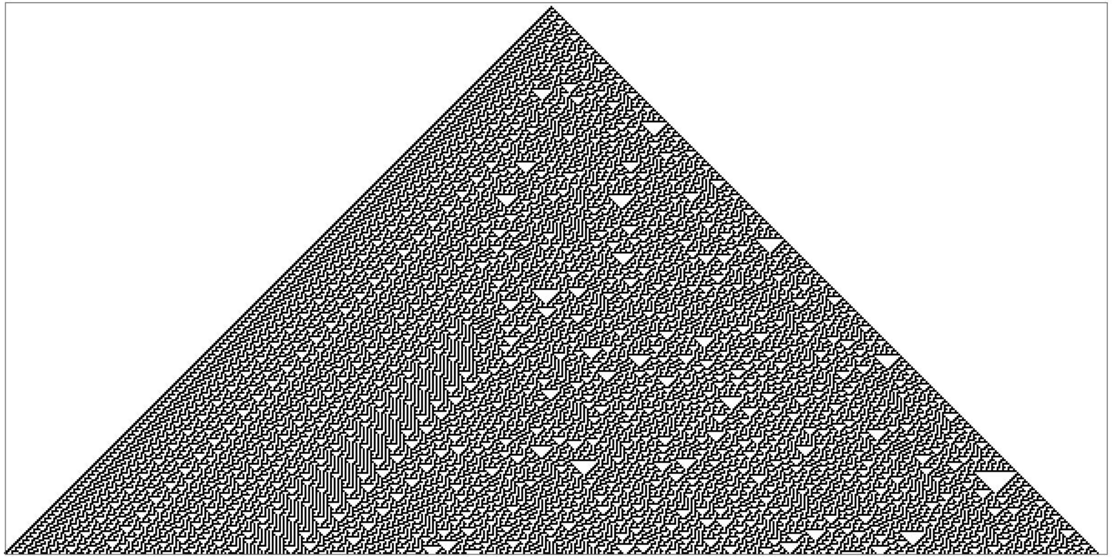


Image credit: WolframAlpha

Traffic Flow Theory: *Problem of Perspective*

- Microscopic Scale:
  - Every vehicle is considered as an individual
  - ODEs
- Macroscopic Scale:
  - All vehicles on the road are considered
  - Similar to models of fluid dynamics
  - PDEs
- Empirical field of study (i.e. recorded data is macro in nature)
- Main Goal: find renormalization approach to connect micro to macro

Traffic Flow Theory: *Parameters of Interest*

- Speed (v)
    - Average of the individual speeds of all of the vehicles in the study area

$$v = \frac{1}{n}\sum_{i=1}^{n} v_i$$

- Density (k)
    - the number of vehicles per unit length of the roadway

$$k = \frac{n}{L}$$

- Flow (q)
    - number of vehicles passing a reference point per unit of time

$$q = kv$$

Traffic Flow Theory: *Speed-Flow-Density Relations*

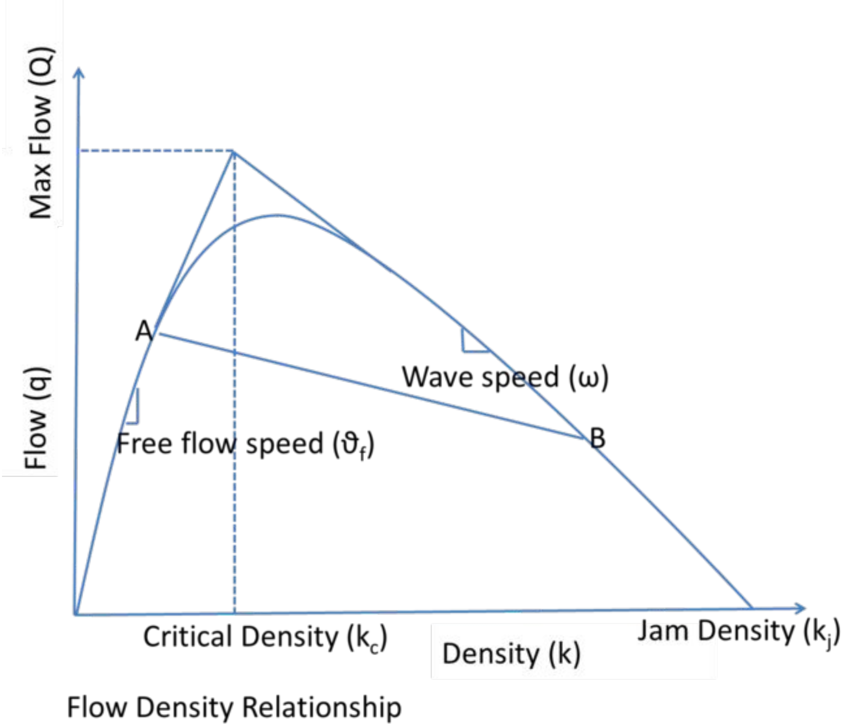
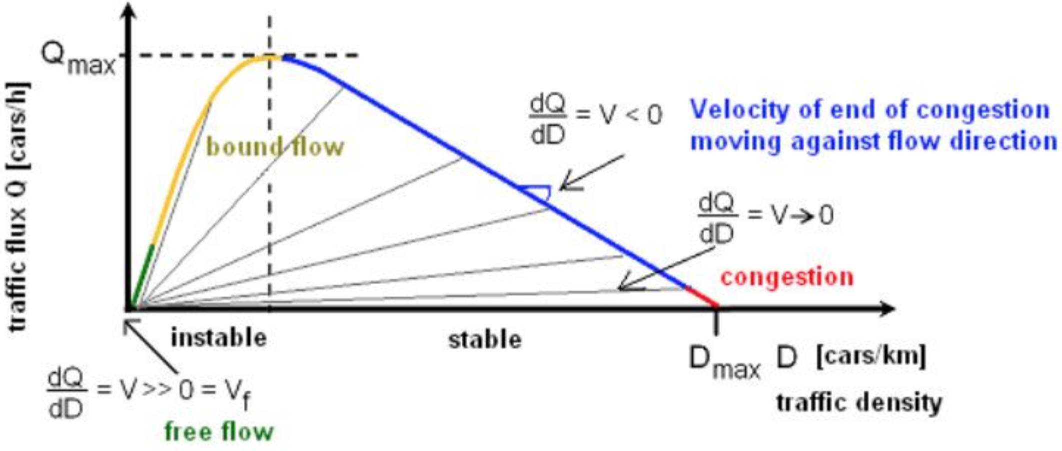
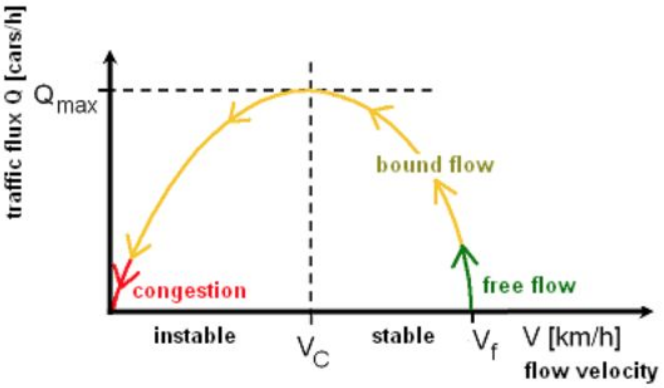
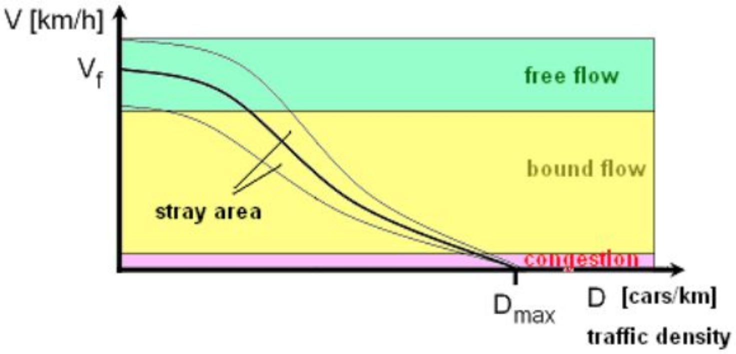


Figure 1: The Fundamental Diagram

Image credit: Wikipedia







- At K_c and V_c, the state of flow will change from stable to unstable.
- If one of the vehicles brakes in unstable flow regime the flow will collapse.
- [Phantom Jams - Ted Ed]()

Figure 2: Stability in Traffic Flow

Image credit: https://www.sciencedirect.com/science/article/abs/pii/S0191261508000180?via%3Dihub

Traffic Flow Theory: *Kinematic Wave Model*

- Simplest and first model to reproduce the capture the behavior of the Fundamental Diagram (i.e. phenomena of traffic waves)
  - Stems from the conservation of vehicles (net flow of cars in and out must be balanced by the change in density)

$$\frac{\partial k}{\partial t} + \frac{\partial q}{\partial x} = 0$$

- Since, $q = kv$

$$\frac{\partial k}{\partial t} + \omega(k)\frac{\partial k}{\partial x} = 0$$

Traffic Flow Theory: *Too Many Micromodels*

- Lighthill-Whitham-Richards (LWR)
- Greenshield
- General Motors (GM)
- Gipps' model
- Payne's model
- Newell's Optimal Velocity (OV) model
- Wiedemann's model
- Whitham's model
- Bando Model
- Treiber’s Intelligent Driver Model (IDM)
- Aw-Rascle model
- …
- **Nagel-Schreckenberg (NaSch) Cellular Automaton (CA) model**

Nagel-Schreckenberg Cellular Automata (NaShc-CA) Model

- A simple stochastic cellular automaton model for road traffic flow that can reproduce traffic jams
- Traffic jams → as an emergent phenomenon due to interactions between cars on the road
- Two Models: *Freeway Traffic* and *Two Lane Traffic*

Reference: https://hal.archives-ouvertes.fr/jpa-00246697/document, https://arxiv.org/pdf/cond-mat/9512119.pdf

NaShc-CA for Freeway Traffic Model

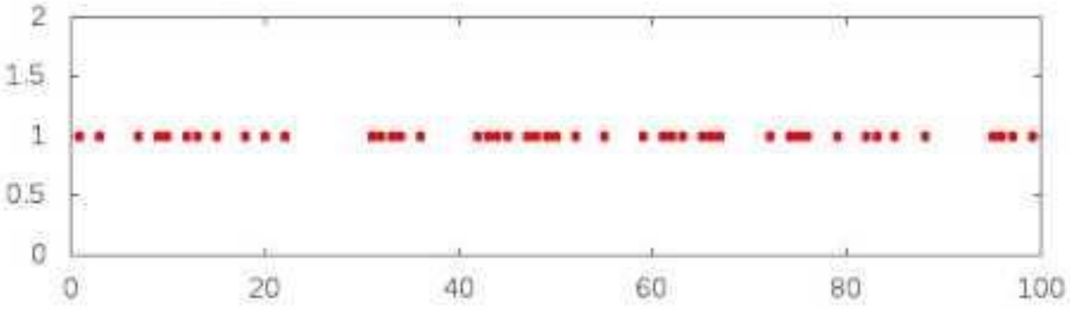
- Lane = 1 D lattice of L cells
- Each cell is either occupied by one of N cars or is empty (i.e. 1 or 0)
- A car can move with the velocity (v) determined by integer values less than some speed limit (v_{max})
- At time t a car (i) is identified by coordinates (cell number, speed) [i.e. ($x^{(t)}_i$, $v^{(t)}_i$)]
- The number of empty cells in front the car is called a gap, $g_i^{(t)} = x_{i+1}^{(t)} - x_i^{(t)} - 1$.
- Each car has a braking probability, p
- The cars move along the lane according to the following rules (Rule 184):

$$
\begin{aligned}
&\text{— } \textit{acceleration: } v_i^{(t+1)} = \min(v_i^{(t)} + 1, v_{max});\\
&\text{— } \textit{deceleration: } v_i^{(t+1)} = \min(v_i^{(t)}, g_i^{(t)});\\
&\text{— } \textit{randomization: } \text{if random} < p \text{ then } v_i^{(t+1)} = \max(v_i^{(t)} - 1, 0);\\
&\text{— } \textit{movement: } x_i^{(t+1)} = x_i^{(t)} + v_i^{(t+1)}.
\end{aligned}
$$

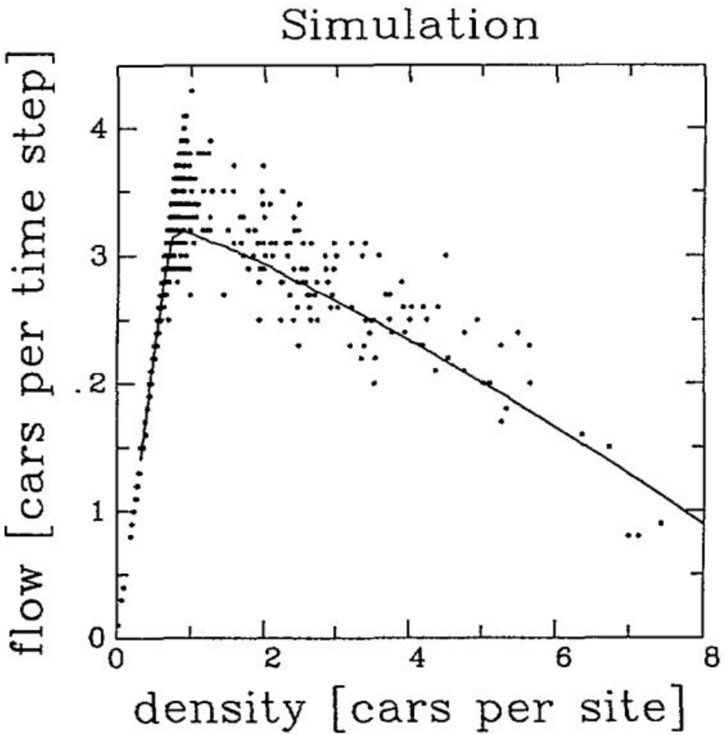
- At each discrete time step t ➔ t + 1 the positions and velocities of all cars are synchronously updated.

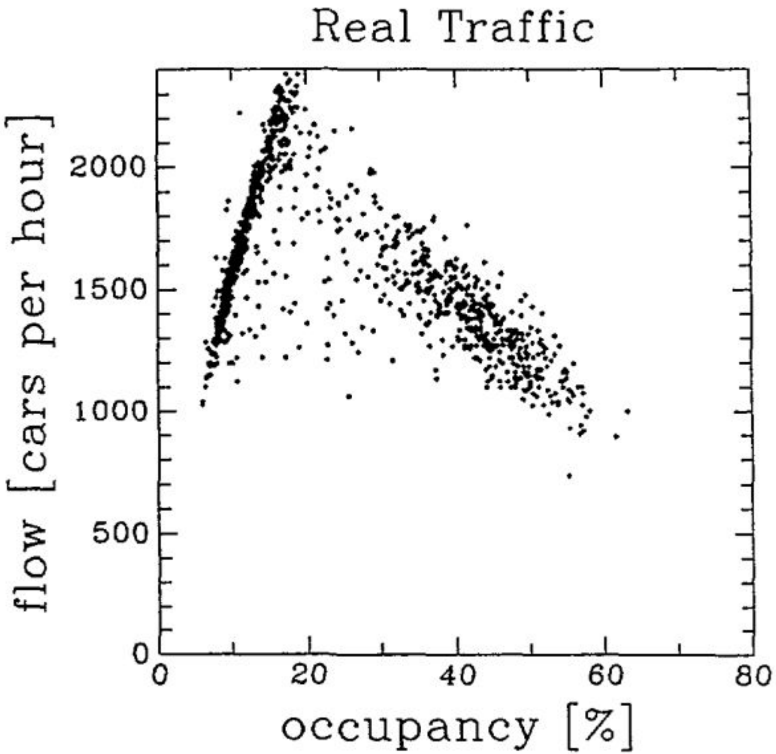
Reference: https://hal.archives-ouvertes.fr/jpa-00246697/document

NaShc-CA for Freeway Traffic Model



NaShc-CA for Freeway Traffic Model





Reference: https://hal.archives-ouvertes.fr/jpa-00246697/document

NaShc-CA for 2-Lane Traffic Model

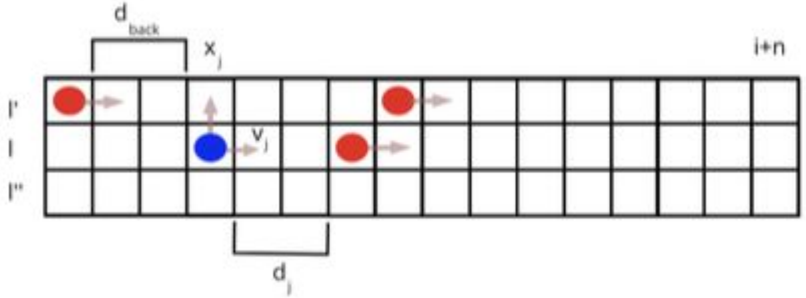


Figure A.11: Road - Car grid in Cellular Automata

Longitudinal Update Rule:

***Rule 1: Acceleration,** $v_j \rightarrow min(v_j + 1, v_{max})$
Rule 2: Braking, $v_j \rightarrow min(v_j, d_j)$
Rule 3: Randomization, $v_j \rightarrow v_j - 1$
(with probability p_s)
Rule 4: Motion, $x_j \rightarrow x_j + v_j$

(a) rules for single-lane highway

Multi-lane Update Rule:

***Rule A: Incentive criterion ,** $v_{l'} > d_j$
***Rule B: Safety criteria ,** $d_{back} > v_{max}$
Rule C: Decision, $l \rightarrow l'$
(with probability p_l)
Rule D: Longitudinal Update Rule

(b) rules for multi-lane highway

Figure 1: Rules for CA models for traffic flow

Reference: https://arxiv.org/pdf/cond-mat/9512119.pdf, [my paper]()

NaShc-CA For Traffic Flow

- The 2 Lane Model Extends to N-lanes
- Heterogeneous Model of Vehicles Classes
  - Intra Vehicle class:
    - Driver personality (e.g. Aggressive, Inattentive, etc) as probabilities
  - Inter Class:
    - Buses, cycles, cars as number of cells taken and v_{max}, lane preference, etc
- Infrastructure Model
  - Bottlenecks (e.g. potholes, speed bumps, lane blockage, etc) as designated cells on the road with their corresponding v_{max}
  - Traffic Lights as cells that vehicles cannot occupy at given time intervals set at certain frequencies
- In summary, it models complex traffic behaviours using simple rules
- Show some simulations

NaShc-CA Model for Autonomous Vehicle (AV) Design

- Opportunistic Model

$$P(braking) = 0; P(lane\ change) = 1;\ v^{av}(t+1) = min(v^{av}(t) + 1, s_{lv}, v^{av}_{max}, v_{sl})$$

- Neighbor Aware Model

$$v^{L}_{max} = \begin{cases} v_{aa}\,, if\ AV - AV \\ v_{ah},\ if\ AV - HV \\ v_{h}\,, if\ HV \end{cases}$$

$$v_{aa} > v_{ah} \geq v_{h}$$

$$v^{av}(t+1) = min(v^{av}(t) + 1, s_{lv}, v^{L}_{max}, v_{sl}) - \delta(P(braking)); \delta(P(braking)) = [0, 1]$$

- Neighbor Aware and Opportunistic Model

$$P(braking) = 0; P(lane\ change) = 1;\ v^{av}(t+1) = min(v^{av}(t) + 1, s_{lv}, v^{L}_{max}, v_{sl})$$

Reference: my paper

Self-Organized Phenomena of AVs

- Self-Organized Phenomena in Free Flow (low to critical Density) for Neighbor Aware models of AV
    - Clustering
    - Lane Domination

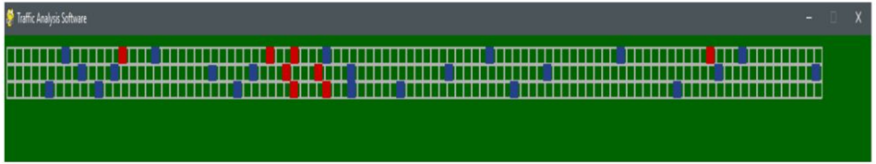


Fig 5: A screen capture of the simulation showing a cluster of 6 AVs (red). [Periodic Boundaries]

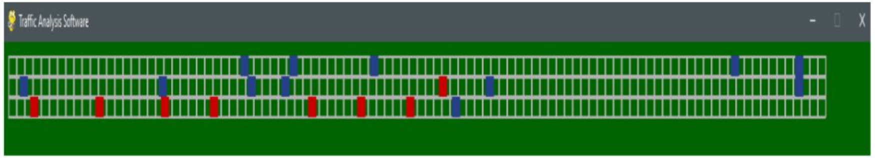


Fig 6: Another screen capture from the simulation showing self-organized lane of 7 AVs (red) .[Periodic Boundaries]

- Clustering is not a random phenomenon but is unique to systems involving opportunistic intelligent agents that are capable of recognition

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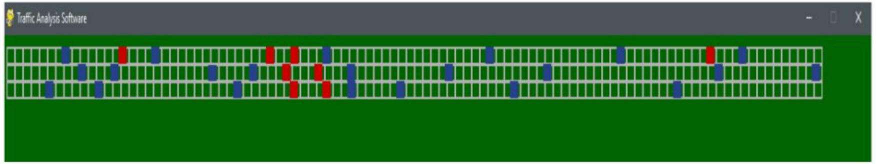

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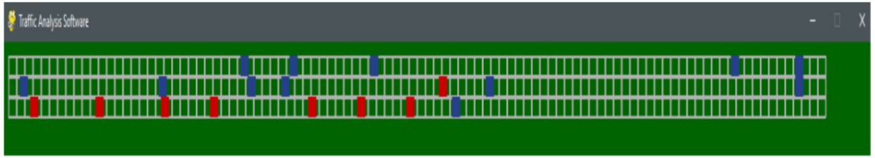

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Self-Organized Phenomena of AVs

- Self-Organization improves traffic at all densities
- Lane Change Dynamics highlights another critical behaviour of AVs
  - AVs are shepherd dogs and HVs are heep

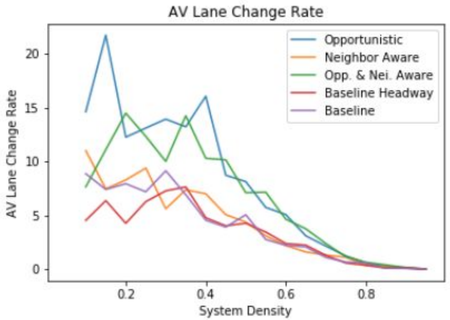

Fig 13: AV Lane Change Rates

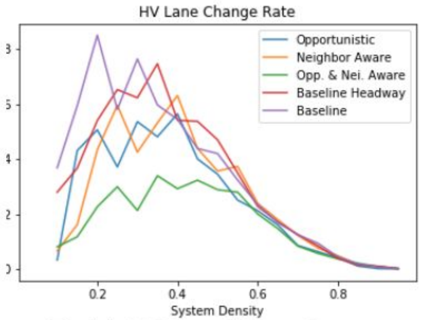

Fig 14: HV Lane Change Rates

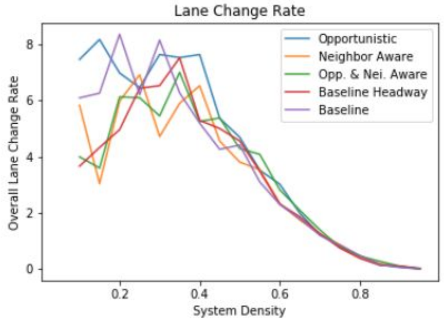

Fig 15: Overall Lane Change Rates

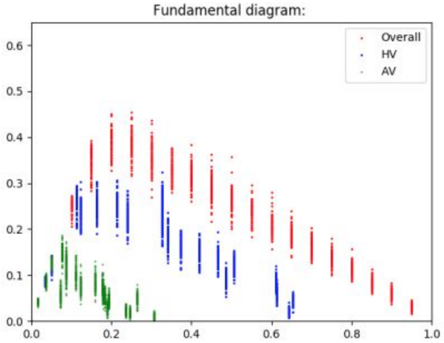

Fig 16: FD of Baseline Model

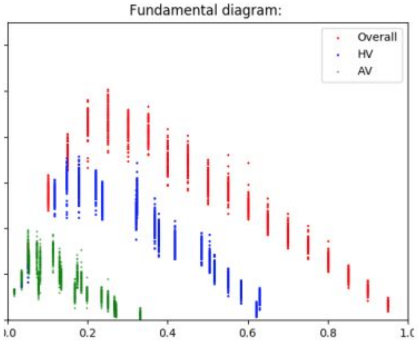

Fig 17: FD of Neighbor Aware and Opportunistic Model

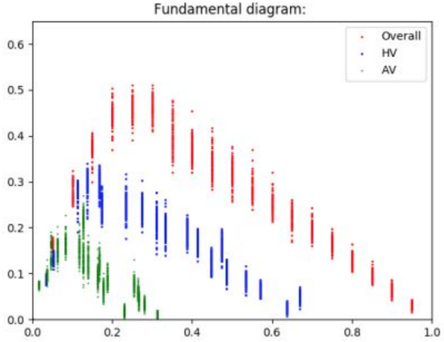

Fig 18: FD of Opportunistic Model

Reference: [my paper]